Problem 1) $f(z) = az^3 + bz^2 + cz + d.$

Let $a = a' + ia''$, $b = b' + ib''$, $c = c' + ic''$, and $d = d' + id''$. We also write $z = x + iy$. We'll have:

$$f(z) = (a' + ia'')(x^3 + 3ix^2y - 3xy^2 - iy^3) + (b' + ib'')(x^2 + 2ixy - y^2)$$
$$+ (c' + ic'')(x + iy) + (d' + id'')$$
$$= a'x^3 + 3ia'x^2y - 3a'xy^2 - ia'y^3 + ia''x^3 - 3a''x^2y - 3ia''xy^2 + a''y^3$$
$$+ b'x^2 + 2ib'xy - b'y^2 + ib''x^2 - 2b''xy - ib''y^2$$
$$+ c'x + ic'y + ic''x - c''y + d' + id''$$

Since $f(z) = u(x,y) + iv(x,y)$, we may write:

$$\begin{cases} u(x,y) = a'x^3 - 3a'xy^2 - 3a''x^2y + a''y^3 + b'x^2 - b'y^2 - 2b''xy + c'x - c''y + d' \\ v(x,y) = 3a'x^2y - a'y^3 + a''x^3 - 3a''xy^2 + 2b'xy + b''x^2 - b''y^2 + c'y + c''x + d'' \end{cases}$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = 3a'x^2 - 3a'y^2 - 6a''xy + 2b'x - 2b''y + c' \\ \dfrac{\partial v}{\partial y} = 3a'x^2 - 3a'y^2 - 6a''xy + 2b'x - 2b''y + c' \end{cases} \Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\begin{cases} \dfrac{\partial u}{\partial y} = -6a'xy - 3a''x^2 + 3a''y^2 - 2b'y - 2b''x - c'' \\ \dfrac{\partial v}{\partial x} = 6a'xy + 3a''x^2 - 3a''y^2 + 2b'y + 2b''x + c'' \end{cases} \Rightarrow \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

The Cauchy-Riemann Conditions are therefore satisfied.